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Tugas I Matematika 2

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1). a. $\ln(1/x) + \ln 2x^3 = \ln 3$

$= \ln x^{-1} + \ln 2 + \ln x^3 = \ln 3$

$-\ln x + \ln 2 + 3\ln x = \ln 3$

$2\ln x = \ln 3 - \ln 2$

$\ln x^2 = \ln \frac{3}{2}$

$x^2 = \frac{3}{2}$

$x = \pm\sqrt{\frac{3}{2}}$

Jawab : $x = \sqrt{\frac{3}{2}}$ atau $x = -\sqrt{\frac{3}{2}}$

b. $\frac{e^x}{1-e^x} = 2$

$e^x = 2 - 2e^x$

$3e^x = 2$

$e^x = \frac{2}{3}$

$\ln e^x = \ln \frac{2}{3}$

$x \ln e = \ln \frac{2}{3}$

$x = \ln 2 - \ln 3$

Jawab : $x = \ln 2 - \ln 3$

2) a) $\frac{d}{dx}\left[\ln \sqrt{\frac{\sqrt{x^3}\sqrt{x+1}}{\sin x \sec x}}\right]$

$= \frac{d}{dx}\left[\ln \left(\frac{x^{3/2}\cdot (x+1)^{1/2}}{\sin x \sec x}\right)^{1/2}\right]$

$= \frac{d}{dx}\left[\ln \frac{x^{3/4}(x+1)^{1/4}}{\sin^{1/2}x \cdot \sec^{1/2}x}\right]$

$= \frac{d}{dx}\left[\ln x^{3/4} + \ln(x+1)^{1/4} - \ln \sin^{1/2}x - \ln \sec^{1/2}x\right]$

$= \frac{d}{dx}\left[\frac{3}{4}\ln x + \frac{1}{4}\ln(x+1) - \frac{1}{2}\ln \sin x - \frac{1}{2}\ln \sec x\right]$

$= \frac{3}{4x}\cdot 1 + \frac{1}{4(x+1)}\cdot 1 - \frac{1}{2}\cdot\frac{1}{\sin x}\cdot\cos x - \frac{1}{2}\cdot\frac{1}{\sec x}\sec x \tan x$

$= \frac{3}{4x} + \frac{1}{4(x+1)} - \frac{\cot an x}{2} - \frac{\tan x}{2}$

$= \frac{3}{4x} + \frac{1}{4(x+1)} - \frac{1}{2}(\cot an x + \tan x)$

b). $\int_{-\ln x}^{\ln x} \frac{e^x}{e^x+4}\,dx$

misal : $u = e^x + 4$

$du = e^x dx$

Batas atas = $u = e^x + 4$

$= e^{\ln x} + 4$

$= x + 4$

Batas bawah = $u = e^x + 4$

$= e^{-\ln x} + 4$

$= e^{\ln x^{-1}} + 4$

$= \frac{1}{x} + 4$

$\rightarrow \int_{\frac{1}{x}+4}^{x+4} u^{-1}\,du = \ln u \Big]_{\frac{1}{x}+4}^{x+4}$

$= \ln x+4 - \ln \frac{1}{x} + 4$

$= \ln \frac{x+4}{\frac{1}{x}+4}$

3) a) $f(x) = 1 - \ln 3x$

$y = 1 - \ln 3x$

$y - 1 = -\ln 3x$

$\ln 3x = 1 - y$

$3x = e^{1-y}$

$x = \frac{e^{1-y}}{3}$ → $f^{-1}(x) = \frac{e^{1-x}}{3}$

Syarat sebuah fungsi punya Invers

$f^{-1}(f(x)) = x$ dan $f(f^{-1}(x)) = x$

Pembuktian :

$f^{-1}(f(x)) = \frac{e^{1-f(x)}}{3}$

$= \frac{e^{1-(1-\ln 3x)}}{3}$

$= \frac{e^{\ln 3x}}{3}$

$= \frac{3x}{3}$

$= x$

$f(f^{-1}(x)) = 1 - \ln 3 f(x)$

$= 1 - \ln 3\left(\frac{e^{1-x}}{3}\right)$

$= 1 - \ln e^{1-x}$

$= 1 - (1-x) \cdot \ln e$

$= x$

terbukti

Domain : $x \in R$

b. $f(x) = \begin{cases} 2x, & x \le 0 \\ x^2, & x > 0 \end{cases}$

- Untuk $x \le 0$

$f(x) = 2x$

$y = 2x$

$x = \frac{y}{2}$ → $f^{-1}(x) = \frac{x}{2}$

(memenuhi, dan ada inversnya, karena $y = 2x$ adalah persamaan garis linear. Jadi, ketika ada garis horizontal hanya memotong max. 1 titik)

- Untuk $x > 0$

$f(x) = x^2$

$y = x^2$

$x = \sqrt{y}$ → $f^{-1}(x) = \sqrt{x}$

Grafik x^2 hanya akan punya invers jika domain dibatasi $x \le 0$ atau $x \ge 0$ karena domainnya $x > 0$, maka $f(x) = x^2$ punya invers. Domain $x \in R$

4). $F(x) = f(2g(x))$. $f(x) = x^4 + x^3 + 1$ untuk $0 \le x \le 2$ dan $g(x) = f^{-1}(x)$.

$F'(3) = \ldots$

$f(x) = 3$ (agar dapat $f^{-1}(3)$)

$x^4 + x^3 + 1 = 3$

$x^4 + x^3 - 2 = 0$

1	1	1	0	0	−2
		1	2	2	2
	1	2	2	2	0

↓ Tak sisa

Jadi $x = 1$ adalah akar $x^4 + x^3 - 2 = 0$

Sehingga : $f(1) = 3$

$f^{-1}(3) = 1 = g(3)$

$F'(x) = f'(2g(x)) \cdot 2 \cdot g'(x)$

$\rightarrow f'(x) = 4x^3 + 3x^2$, $0 \le x \le 2$

$g'(3) = f^{-1}{}'(3) = p$

$f'(p) = 3$

$4p^3 + 3p^2 = 3$

$p \approx 0{,}715$ (memenuhi daerah asal)

→ $g'(3) \approx 0{,}715$

$F'(3) = f'(2g(3)) \cdot 2g'(3)$

$= f'(2 \cdot 1) \cdot 2(0{,}715)$

$= (4(2)^3 + 3(2)^2) \cdot 1{,}43$

$= (32 + 12) \cdot 1{,}43$

$= (44)(1{,}43)$

$= 62{,}92$